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Entrainment rates in turbulent shear flows

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The rate of entrainment of ambient fluid across a turbulent interface has been defined as the mean rate of increase of turbulent fluid in the flow direction. Experiments to measure this quantity by conditional sampling in a two-dimensional wall jet are described. Further, estimates of this entrainment rate were made for the turbulent boundary layer, two-dimensional wake, two-dimensional jet and round jet and the results are discussed.

1. Introduction

This investigation is concerned with the phenomenon of entrainment, which is a feature of all free turbulent shear flows. The very rapid spreading of such flows as wakes and jets (as compared with their laminar counterparts) is ascribed to the assimilation of the surrounding non-turbulent fluid by the turbulence. The exact mechanism of this process is not fully understood. It is generally accepted that the originally irrotational fluid acquires vorticity by viscous diffusion (there is no other way) and this vorticity is amplified by the rate-of-strain field (see Phillips 1972). It can be argued further that this process must lead to a sharp interface which separates the turbulent and non-turbulent regions. The existence of such an interface is well known. It can be treated as a front which propagates into the irrotational field.

It will be assumed that the interface is a continuous surface whose location is a random function of space and time. The aim will be to define a meaningful entrainment rate and to relate this to the global properties of turbulent flows.

2. Experimental details

Properties of the turbulent zone were measured in a two-dimensional turbulent wall jet. The purpose, as is shown below, was to determine the entrainment rate from these properties. The wall jet flow is described in detail by Paizis (1972).

The streamwise velocity $U(t)$ was obtained using Thermo-Systems Model 1274 boundary-layer probes and Thermo-Systems Model 1010 constant-temperature anemometers and linearizers. The probes were calibrated against a Pitot tube in a low turbulence wind tunnel. The intermittency function $I(t)$ was constructed by suitably processing the anemometer output in a turbulence detector circuit (Paizis & Schwarz 1974).

Conventional mean velocities were measured by using a DYMEC Model 2210

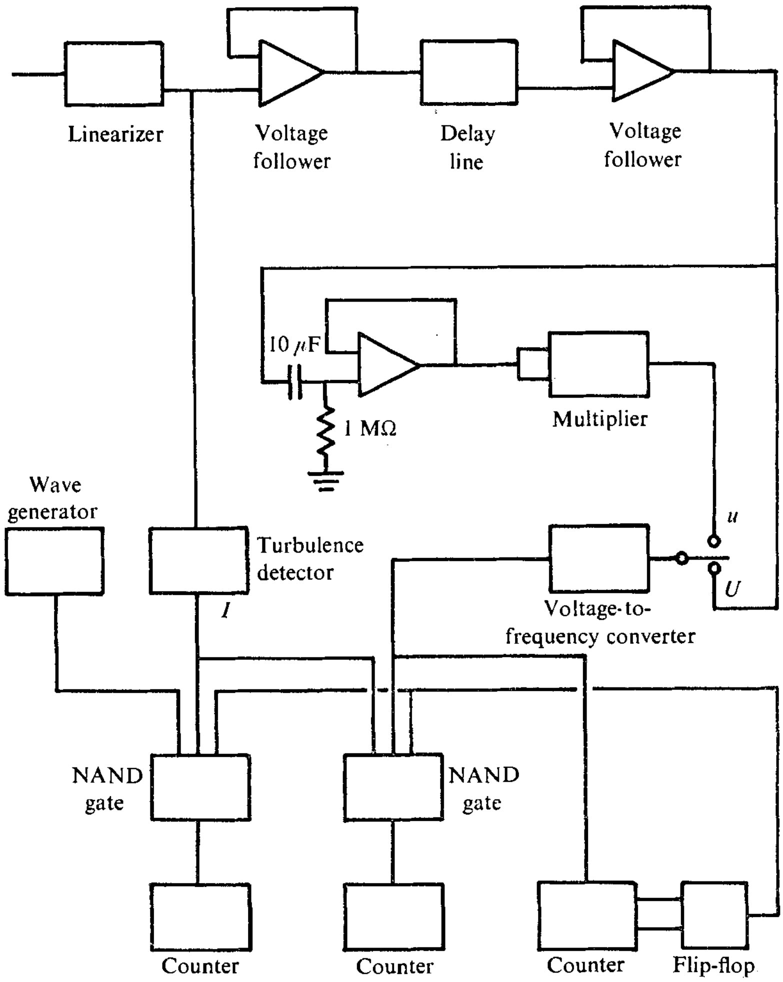

FIGURE 1. Zone-average measurement arrangement.

voltage-to-frequency converter coupled with a Beckman 6148 EPUT and timer. The signal from the linearizer formed the input to the voltage-to-frequency converter, whose output is a pulse train with frequency proportional to the amplitude of the incoming signal. The number of pulses was then counted over a fixed period (usually 100 s) by the Beckman counter. For the measurement of intensities, the d.c. component of the linearizer output was blocked using a high-pass RC filter. The resulting signal was squared by an Analog Devices 426A multiplier and the mean value of the multiplier output measured as above.

Turbulent zone averages were measured by gating the voltage-to-frequency output with the intermittency signal. In addition a delay of the linearizer output was necessary in order to match the delayed intermittency signal. This was achieved using an Ad-Yu 802 G delay line. The intermittency signal was formed from the velocity signal before delay. Three counters were used so that $\overline{U}$, $\overline{UI}$ and $\overline{I}$ (or $\overline{u^2}$, $\overline{u^2I}$ and $\overline{I}$) could be measured simultaneously, thereby reducing errors considerably. Since only one of the counters was equipped to count automatically for 100 s, its timing pulses ('start' and 'stop') were used to gate the inputs to the other two counters. The gating signal was obtained from a flip-flop circuit which was controlled by the 'start' and 'stop' pulses.

The intermittency factor was measured in the conventional manner, by gating a high frequency wave (100 kHz) with the intermittency signal and counting the resulting pulse train. The arrangement described in this section is illustrated in figure 1.

3. A measure of entrainment

In order to relate the entrainment to other properties of the turbulent flow it is necessary to have a rigorous definition of the entrainment rate. In any turbulent flow, the amount of turbulent fluid crossing any semi-infinite plane perpendicular to the x direction per unit transverse width is

$$\int_0^\infty UI\,dy,$$

where U is the x component of velocity and I is the intermittency function, which is unity at any point if the flow at that point is turbulent and zero otherwise. The mean rate of increase with x of the above quantity defines the entrainment rate $\mathscr{U}_y$:

$$\mathscr{U}_y = \frac{d}{dx}\overline{\int_0^\infty UI\,dy}. \tag{1}$$

For two-dimensional flows this is the volume of fluid entrained per unit projected area.

For the case where the interface position $Y(x,z,t)$ is a single-valued function of (x,z), this definition is clarified by the following derivation. Let dS be an element of area of the interface whose projection onto the x,z plane is $dx\,dz$. The rate at which fluid crosses this surface element is

$$(\partial Y/\partial t)\,dx\,dz + U(Y)\,dY\,dz - V(Y)\,dx\,dz + W(Y)\,dY\,dx,$$

and its value per unit projected area defines the instantaneous entrainment rate:

$$\mathscr{U}'_y = \frac{\partial Y}{\partial t} + U(Y)\frac{\partial Y}{\partial x} - V(Y) + W(Y)\frac{\partial Y}{\partial z}.$$

A mass balance over the projected volume shows that

$$\mathscr{U}'_y = \frac{d}{dx}\int_0^Y U\,dx + \frac{d}{dz}\int_0^Y W\,dz - V(0).$$

Assuming that the x,z plane is either a wall or a plane of symmetry and that the flow is steady and two-dimensional, this equation becomes, on averaging,

$$\mathscr{U}_y = \overline{U(Y)\frac{\partial Y}{\partial x}} - \overline{V(Y)} = \frac{d}{dx}\int_0^\infty \overline{UI}\,dy.$$

Notice that $\overline{V(Y)}$ is equal to neither $\overline{V}(\infty)$ nor $\overline{V}(\overline{Y})$. The former is true if V is constant in the non-turbulent region while the latter follows if it is assumed that fluctuations in Y are small and that V and Y are uncorrelated. These results follow from the relation

$$\overline{V(Y)} = \overline{\int_0^Y \frac{\partial V}{\partial y}\, dy}.$$

Of course the same can be said for $\overline{U(Y)\,\partial Y/\partial x}$.

Zone-average measurements

The occurrence of the quantity $\overline{UI}$ in the expression for the entrainment rate prompts an investigation of the properties of the turbulent zone. The turbulent zone average of the velocity can be defined as

$$\overline{U}_T = \overline{UI}/\overline{I}, \tag{2}$$

while the non-turbulent zone average is

$$\overline{U}_N = \overline{U(1-I)}/(1-\overline{I}).$$

It follows that
$$\overline{U} = \overline{UI} + \overline{U(1-I)} = \overline{I}\,\overline{U}_T + (1-\overline{I})\,\overline{U}_N. \tag{3}$$

For the mean-square values of the velocity it is desirable to subtract the zone mean value first. Thus

$$\overline{u_T^2} = \frac{\overline{(U-\overline{U}_T)^2 I}}{\overline{I}} = \frac{\overline{u^2 I}}{\overline{I}} - (\overline{U}-\overline{U}_T)^2, \tag{4}$$

where $u = U - \overline{U}$. The non-turbulent mean-square velocity is

$$\overline{u_N^2} = \overline{u^2(1-I)}/(1-\overline{I}) - (\overline{U}-\overline{U}_N)^2.$$

The following formula, relating the three intensities, may be easily derived:

$$\overline{U}^2 - \overline{u^2} = \overline{I}(\overline{U}_T^2 - \overline{u_T^2}) + (1-\overline{I})(\overline{U}_N^2 - \overline{u_N^2}). \tag{5†}$$

The quantities $\overline{U}$, $\overline{UI}$, $\overline{u^2}$, $\overline{u^2 I}$ and $\overline{I}$ were measured in the wall jet at $x/d = 360$, and the turbulent zone averages $\overline{U}_T$ and $\overline{u_T^2}$ were computed from (2) and (4). The non-turbulent zone averages $\overline{U}_N$ and $\overline{u_N^2}$ were then calculated from (2) and (5).

The results are collected in figure 2. Some points are worth noting. At the half-intermittency point the turbulent mean velocity exceeds the conventional velocity by $0{\cdot}07\overline{U}_m$ (where $\overline{U}_m$ is the maximum velocity). The non-turbulent mean velocity becomes significant as the fully turbulent region is approached. Almost all the fluctuations are carried by the turbulent part of the flow. It is not known what the limiting values of the turbulent zone averages should be as the distance from the wall increases, nor to what limit the non-turbulent zone averages tend as the fully turbulent region is approached. Neither does there seem to be any way of predicting these limits.

† The expression (22), relating these quantities, of Kovasznay, Kibens & Blackwelder (1970) is incorrect.

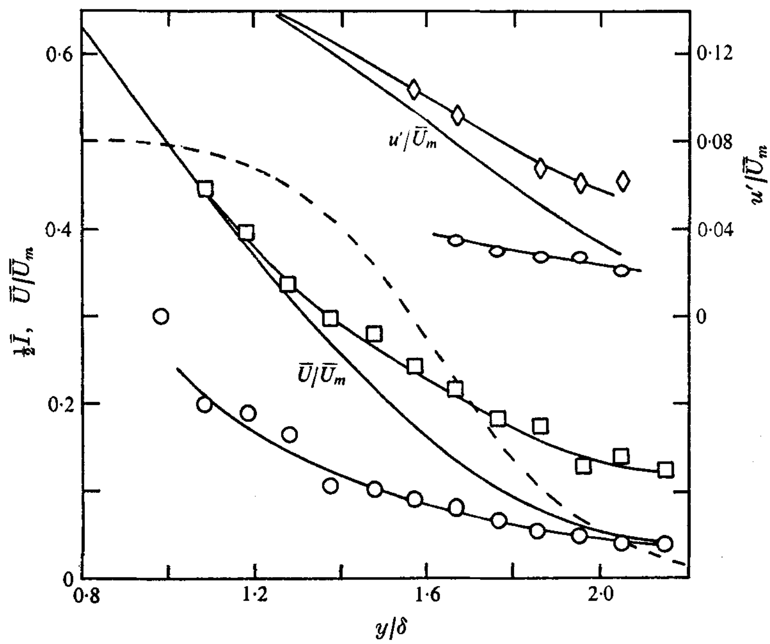

FIGURE 2. Zone-average velocity measurements. △, $\bar{U}_T/\bar{U}_m$; ○, $\bar{U}_N/\bar{U}_m$; ◊, $u'_T/\bar{U}_m$; ⬭, $u'/\bar{U}_m$; ---, $\bar{I}$.

Measurements of the zone mean velocities were repeated at two more downstream stations, $x/d = 90$ and 180. These results are shown in figure 3, where $\bar{U}_T - \bar{U}$ and $\bar{U}_N - \bar{U}$ are presented. It is clear that the turbulent mean velocity profiles are not similar, so it can be concluded that the intermittent region of the wall jet flow does not have a self-preserving structure. This fact is also reflected in the intermittency profiles for the three stations.

A possible measure of the attainment of a self-preserving structure is the position at which the intermittency factor is 0·5. According to this measure, self-preservation is reached at $x/d = 400$ (Paizis 1972). Interestingly enough, the non-turbulent velocity profiles appear to be self-similar.

The measurements reported here are subject to a number of possible errors and it is worth discussing their significance. At the outer edge of the jet the total mean velocity can differ significantly from the mean longitudinal velocity and the hot-wire probe will respond to this total velocity. A correction can be found by using the continuity equation. Judging by the results of Heskestad (1963), this correction becomes significant for $y > 2\delta$. Wygnanski & Fiedler (1969) point out the need for long averaging times in order to reduce the scatter in measurements in the outer region of the jet. This is even more important for the determination of the zone averages. For example, for the turbulent zone averages the actual time during which the average is determined is the integration period multiplied by the intermittency factor. The non-turbulent averages were obtained indirectly. The experiment could have been repeated with I replaced by $1-I$ to obtain these terms directly and hence more accurately. Finally, for the

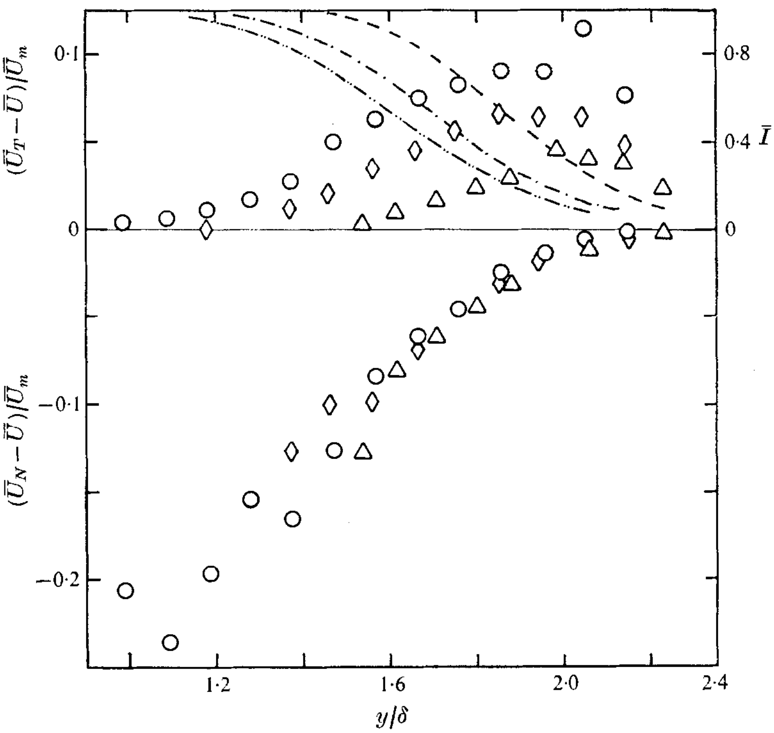

FIGURE 3. Variation of turbulent and non-turbulent mean velocity profiles with downstream distance: △, $x/d = 90$; ◇, $x/d = 180$; ○, $x/d = 360$. ———, $\bar{I}$, $x/d = 90$; —·—·, $\bar{I}$, $x/d = 180$; ——--, $\bar{I}$, $x/d = 360$.

determination of the intensities, the two terms in (3) for $\overline{u_T^2}$ and in (5) for $\overline{u_N^2}$ are of the same order and their subtraction will amplify any errors. The primary aim of this set of measurements was to establish whether $\overline{UI}$ could be replaced by $\overline{U}$ in the expression for the entrainment rate, so no attempt has been made to correct for these various sources of error.

Measurement of entrainment rates

The expression (1) for the entrainment rate can be written as

$$\mathscr{U}_y = \frac{d}{dx}\int_0^\infty \overline{U}\,dy - \frac{d}{dx}\int_0^\infty (\overline{U} - \overline{UI})\,dy, \tag{6}$$

and it will be shown experimentally that for the wall jet the second term can be neglected. For the self-preserving wall jet the maximum velocity $\overline{U}_m$ and the boundary-layer width δ behave respectively as (Schwarz & Cosart 1961)

$$\overline{U}_m = b(x-x_0)^{-\beta}, \quad \delta = a(x-x_0),$$

where a, b, β and x_0 are constants. Also, the integral

$$J_1 = \int_0^\infty \frac{\overline{U}}{\overline{U}_m}\,d(y/\delta)$$

of the mean velocity is independent of x.

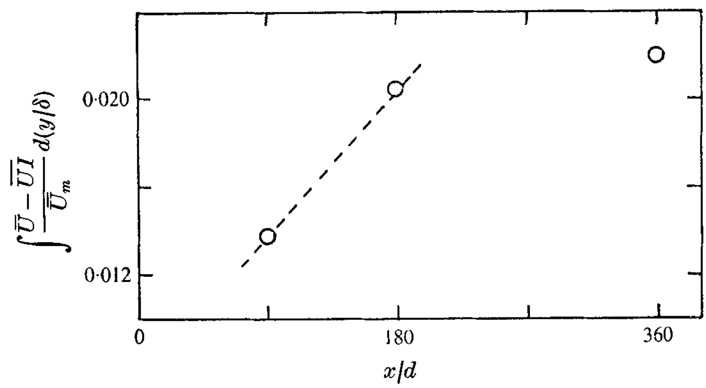

FIGURE 4. Variation of volume of turbulent fluid with downstream distance.

Substituting these forms into (6) gives

$$\frac{\mathscr{U}_y}{\overline{U}_m} = \frac{J_1}{\overline{U}_m}\frac{d}{dx}(\overline{U}_m\delta) - \frac{1}{\overline{U}_m}\frac{d}{dx}\left[\overline{U}_m\delta\int_0^\infty \frac{\overline{U}-\overline{UI}}{\overline{U}_m}d\left(\frac{y}{\delta}\right)\right]$$

$$= J_1 a(1-\beta) - a(1-\beta)\int_0^\infty \frac{\overline{U}-\overline{UI}}{\overline{U}_m}d\left(\frac{y}{\delta}\right) - a(x-x_0)\frac{d}{dx}\int_0^\infty \frac{\overline{U}-\overline{UI}}{\overline{U}_m}d\left(\frac{y}{\delta}\right).$$

The integral occurring in this expression was calculated for the three downstream stations mentioned previously and the results are given in figure 4. For the integral, the largest value over the range of measurement occurs at $x/d = 360$ and is 0·022, while for its derivative, this occurs at $x/d = 180$ (taking the slope as the straight line joining the first two points) and is $0{\cdot}63 \times 10^{-4}$. Thus

$$\mathscr{U}_y/\overline{U}_m = J_1 a(1-\beta) - 0{\cdot}022a(1-\beta) - 0{\cdot}011a.$$

Since J_1 and β are approximately 1·1 and $\frac{1}{2}$ respectively, the error involved in neglecting the last two terms will be 4 % or less.

Replacing $\overline{UI}$ with $\overline{U}$ amounts to setting the entrainment velocity equal to the normal velocity at infinity. The term neglected gives the difference between these two velocities and accounts for the longitudinal motion induced in the non-turbulent region. Since

$$(\overline{V}(\infty) - \mathscr{U}_y)/\overline{V}(\infty) = 0{\cdot}04,$$

this represents a small fraction of the normal velocity at infinity.

Entrainment rates in self-preserving flows

To calculate the entrainment rate for other flows, knowledge of the variation of $\overline{UI}$ with downstream distance is needed. Unfortunately, there seem to be no such data published for any flows. Consequently some approximations must be made, and for this purpose it will be convenient to consider only approximately self-preserving flows.

Boundary layers. An inspection of the zone-average measurements of Kovasznay *et al.* (1970) suggests that a reasonable assumption for the boundary layer might be that the mean velocity in the non-turbulent region is constant and so equal to the free-stream velocity: $\overline{U}_N = \overline{U}_\infty$. With this assumption,

$$\overline{UI} = \overline{U} - \overline{U}_\infty(1-\overline{I}) = \overline{U}_\infty \overline{I} - (\overline{U}_\infty - \overline{U}).$$

The entrainment rate reduces to

$$\mathscr{U}_y = \frac{d}{dx}\int_0^\infty \overline{U}_\infty \overline{I}\, dy - \frac{d}{dx}\int_0^\infty (\overline{U}_\infty - \overline{U})\, dy$$

$$= \overline{U}_\infty \left(\frac{d\overline{Y}}{dx} - \frac{d\delta^*}{dx}\right), \qquad (7)$$

where δ^* is the displacement thickness of the boundary layer. This result was found by Corrsin & Kistler (1955). From (7), for the range of downstream distance covered in their experiments, the ratio of the entrainment velocity to the free-stream velocity decreased from about 0·015 to 0·011. By assuming that all length scales are proportional, the entrainment velocity for the experiments of Kovasznay *et al.* (1970) is estimated to be $0{\cdot}012\,\overline{U}_\infty$.

The more usual form for the entrainment velocity, apparently first derived by Head (1958), in effect assumes that the interface is steady and located at $y = \delta$, the boundary-layer thickness, and that the mean velocity for $y > \delta$ is constant. The result is that $\overline{Y}$ is replaced by δ in (7). For the measurements of Corrsin & Kistler (1955) this formula gives an entrainment velocity ranging from $0{\cdot}0188\,\overline{U}_\infty$ to $0{\cdot}0114\,\overline{U}_\infty$, while for those of Kovasznay *et al.* (1970) it gives $0{\cdot}016\,\overline{U}_\infty$.

In applying his kinematical analysis of the interface to the results of Kovasznay *et al.* (1970), Phillips (1972) estimated that the propagation velocity of the interface should be $0{\cdot}12\,\overline{U}_\infty$. The propagation velocity is equal to the entrainment rate per unit area of surface, so expressed per unit projected area the value would be even higher. A probable explanation for this large discrepancy in entrainment rates is that the prediction of Phillips is for those large-scale motions which cause rapid entrainment. It is highly unlikely that these motions occur continuously, so the propagation velocity of Phillips represents a biased average, conditioned on the existence of a large entrainment rate.

Two-dimensional wakes. Townsend (1956, p. 163) presented some zone-average measurements of U in a wake and concluded that the turbulent mean velocity was approximately equal to the mean velocity. This would imply that U and I are uncorrelated. However, from his measurements it seems that an assumption equivalent to that made for a boundary layer, viz. $\overline{U}_N = \overline{U}_\infty$, would be equally appropriate. Both assumptions lead to the same result but since the latter yields the result more easily, it will be used. The velocity defect for a wake can be written as

$$\overline{U}_\infty - \overline{U} = U_0 f(y/\delta),$$

where U_0 is the velocity scale, usually the difference between the centre-line velocity and $\overline{U}_\infty$, δ is a length scale and f is a universal function independent of x. Substituting this into (6) yields

$$\mathscr{U}_y = \frac{d}{dx}\int_0^\infty \overline{U}_\infty \overline{I}\, dy - \frac{d}{dx}(U_0\delta)\int_0^\infty f\left(\frac{y}{\delta}\right) d\left(\frac{y}{\delta}\right).$$

The second integral is independent of x, and since the two-dimensional wake is a constant Reynolds number flow, i.e. $U_0\delta$ = constant, the second term is zero, hence

$$\mathscr{U}_y = \overline{U}_\infty d\overline{Y}/dx.$$

For the self-preserving wake, the following results hold (Townsend 1956, p. 137):

$$U_0/\overline{U}_\infty = b'(x-x_0)^{-\frac{1}{2}}, \quad \delta = a'(x-x_0)^{\frac{1}{2}}.$$

All lengths are taken to be non-dimensionalized with the cylinder diameter. The entrainment velocity is then

$$\mathscr{U}_y = \tfrac{1}{2}\overline{U}_\infty a'(x-x_0)^{-\frac{1}{2}},$$

and relative to U_0,

$$\mathscr{U}_y/U_0 = a'/2b'.$$

Values for b' are available (e.g. Townsend 1956, p. 135) while a' can be estimated from Townsend's results as presented by Corrsin & Kistler (1955). With $a' = 0{\cdot}33$ and $b' = 0{\cdot}1$,

$$\mathscr{U}_y/U_0 = 1{\cdot}7.$$

Two-dimensional jets. According to the results of the previous section, for the wall jet and analogously for the free jet, the assumption that $\overline{UI} = \overline{U}$ is satisfactory. This means that the non-turbulent velocity is constant and hence zero. Thus the assumption is equivalent to those made before. For self-preservation of the flow,

$$\delta = a(x-x_0), \quad U_0 = b(x-x_0)^{-\beta}. \tag{8), (9}$$

All lengths here are normalized with the jet slot width. For the free jet $\beta = \frac{1}{2}$, while this same value has been found to agree with the experimental results for wall jets (Schwarz & Cosart 1961). Our measurements (Paizis 1972) indicate that $\beta \doteq \frac{1}{2}$. The mean velocity can be written as

$$\overline{U} = U_0 f(y/\delta),$$

and the entrainment velocity becomes

$$\mathscr{U}_y = J_1 d(U_0\delta)/dx, \tag{10}$$

where J_1 is approximately 1·1 (Kohan 1968; Bradbury 1967). Thus (10) becomes, on substituting (8) and (9),

$$\mathscr{U}_y = 0{\cdot}55ab(x-x_0)^{-\frac{1}{2}},$$

and

$$\mathscr{U}_y/U_0 = 0{\cdot}55a.$$

For wall jets $a = 0{\cdot}086$ (Paizis 1972), so

$$\mathscr{U}_y/U_0 = 0{\cdot}047,$$

while for free jets $a = 0{\cdot}1$ (Kohan 1968) and

$$\mathscr{U}_y/U_0 = 0{\cdot}055.$$

Round jets. For axisymmetric flows it is necessary to redefine the entrainment rate. A preferable alternative is to define the entrainment rate per unit axial length as

$$Q_r = \pi\frac{d}{dx}\int_0^\infty \overline{U_x I}\, r\, dr.$$

For the single-valued case, if $R(x, \theta, t)$ is the location of the interface,

$$Q_r = \pi \overline{R(\partial R/\partial t - U_r(R) + U_x \partial R/\partial x)}.$$

Note that this is the entrainment rate for the semi-infinite region $0 \leqslant \theta \leqslant \pi$. The total entrainment is twice the quantity defined above.

The self-preserving relations for a round jet are

$$U_0 = b(x-x_0)^{-1}, \quad \delta = a(x-x_0).$$

Using these, together with the assumption that $\overline{U_x I} = \overline{U}_x$, we find

$$Q_r = \pi J_1' a^2 b,$$

where

$$J_1' = \int_0^\infty \frac{\overline{U}_x r}{U_0 \delta} d\left(\frac{r}{\delta}\right) = 0{\cdot}63$$

(Bradbury 1967). Normalizing with U_0 and δ and using $a = 0{\cdot}085$ (Townsend 1956, p. 184) yields

$$Q_r/U_0\delta = \pi J_1' a = 0{\cdot}17.$$

Comparison of entrainment rates

It is difficult to compare the entrainment rates for the various flows. Certainly the boundary layer does have extremely small entrainment rates. However, the large value for the two-dimensional wake is misleading for it is mostly a result of the small velocity scale used to normalize the entrainment velocity. On an absolute basis, i.e. if the free-stream velocity of the wake is equal to the exit velocity of the two-dimensional wall jet, the entrainment velocities are about equal. For the wake

$$\mathscr{U}_y = 0{\cdot}17\overline{U}_\infty(x-x_0)^{-\frac{1}{2}},$$

while for the wall jet, using $b = 3{\cdot}7U_1$, where U_1 is the jet exit velocity (Paizis 1972),

$$\mathscr{U}_y = 0{\cdot}18U_1(x-x_0)^{-\frac{1}{2}}.$$

The free jet gives similar results but the round jet differs. In fact, using $b = 5U_1$ (Wygnanski & Fiedler 1969) we find

$$Q_r = 0{\cdot}07U_1 d,$$

where d is the jet diameter. Thus the entrainment rate is constant.

Negative and zero entrainment

An intriguing question that emerges in this problem is whether or not the entrainment rate can be negative or zero. Since the generally accepted mechanism by which entrainment occurs is through the diffusion of vorticity to previously irrotational fluid, it seems that the entrainment rate can only be positive. However, as Moffatt (1965) observed, the dissipation of vorticity proceeds at the same rate, so unless there exists a rate-of-strain field to amplify the newly acquired vorticity, it will be dissipated and the entrainment rate will be effectively zero. This was demonstrated by Mobbs (1967) in what was essentially a zero momentum-deficit wake. He found that the mean width of the turbulent fluid actually decreased with downstream distance. Assuming that U and I were uncorrelated (or equivalently, $\overline{U}_N = \overline{U}_\infty$, as for self-preserving wakes) it follows that the rate of entrainment was negative. This can only be ascribed to the decay of the turbulent field, perhaps enhanced in some way by the large fluctuations in the interface location.

Another type of flow usually cited as an example of the occurrence of negative entrainment is the boundary layer in a favourable pressure gradient, where 'relaminarization' takes place. Kovasznay (1971) has the pointed out that recent measurements (Blackwelder & Kovaszay 1972) show this to be a misnomer, the large increase in mean velocity accounting for the decay in the relative turbulence levels.

This question can be resolved unequivocally by measurements of the variation of $\overline{UI}$ with x in the above-mentioned flows.

4. Conclusions

A rigorous definition of the entrainment rate of turbulent flows has been proposed as the mean of the instantaneous rate of increase of turbulent fluid with the downstream distance. This determines the mean rate of conversion of irrotational ambient fluid into turbulent fluid through the interface. The conventional definition of an entrainment rate has the form of the product of a characteristic velocity scale with the growth rate of a characteristic length scale, and may be related to the inflow of ambient fluid towards the turbulent flow. This definition is meaningless for flows that are not self-preserving and crude for self-preserving flows. Further, not all of this inflow is converted into turbulent fluid and a portion is 'dragged along' by the turbulent interface. Therefore, the two definitions are fundamentally different and a distinction must be made if comparisons are to be made with theories that predict the propagation of the turbulent interface into non-turbulent fluid.

The proposed entrainment rate can be measured for any turbulent flow using modern conditional sampling techniques and experimental results are given for a two-dimensional turbulent wall jet. Further, by making suitable assumptions about the turbulence properties, the entrainment rate can be expressed in terms of global parameters for various self-preserving flows. For each case considered, the basic assumption was essentially the same, viz., that the non-turbulent mean velocity was constant. It then followed that the entrainment velocity was a self-preserving quantity, and the final expressions were generally similar to the entrainment constant introduced by Townsend (1966). However, the definition of the entrainment rate (1) is different from that of the entrainment constant, which depends on assuming a lack of correlation between flow variables for significance.

In addition, turbulent zone averages have been presented for the turbulent wall jet. It was found that the difference between the turbulent and non-turbulent mean velocities at the half-intermittency point was $0{\cdot}14\overline{U}_m$.

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